Attitude Dynamics of Satellites with Flexible Appendages—A Brief Review

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EXACTING demand on precise orientation of a satellite, relative to an inertial or an orbiting frame of reference, has resulted in a whole new area of scientific endeavor referred to as attitude dynamics. In the early stages of space exploration when spacecraft tended to be small, mechanically simple, and essentially inflexible, the elastic deformations were relatively insignificant. Numerous investigations involving active and passive stabilization procedures and accounting for internal as well as external forces have been carried out assuming satellites to be rigid.[1]* However, in a modern space vehicle carrying lightweight deployable members, which are inherently flexible, this is no longer true. Motivations for this trend are many.

1) Every satellite goes through "a brief interval of vigorous acceleration and vibration during boost, followed by a prolonged functioning in a quiescent mode of operation characterized by extremely small loads and acceleration."[2] The universal solution to this dilemma has been to design large members (solar panels, booms, antennas, etc.) as flexible bodies that can be stored in a compact fashion (and hence behave like rigid bodies) during the launch but emerge like a butterfly from its cocoon after the boost terminates.

2) In response to demanding mission requirements, satellite configurations have become increasingly complex to maintain a delicate balance between diverse constraints of space, weight, performance, control, and their optimum realization. For example, ever-increasing demand on power for operation of the on board instrumentation, scientific experiments, communications systems, etc., has been reflected in the size of the solar panels. It has increased to a point where their flexibility can no longer be neglected. For example, the proposed Canadian Communications Technology Satellite (CTS), to be launched in 1975, is designed to carry two solar panels, 3.75 ft × 24 ft each, to generate 1.2 kw.[3–8]

3) Use of larger members may be essential in some missions. For example, Radio Astronomy Explorer (RAE) Satellite[9,10] used four 750 ft antennas for detecting low-frequency signals.

4) Certain multipurpose missions, particularly those requiring simultaneous relative measurements from points appreciably apart, may find elastically supported or the recently proposed cable connected multibody systems attractive.[11–18]

5) Modular construction leading to a gigantic space station through elastic linkages can no longer be considered a fantasy. In this connection, studies of far-reaching significance by Roberson, Wittenburg, and others are of particular relevance.[19–27]

6) For longer life span, passive or at least semi-passive attitude control procedures are ideal. They normally involve use of long booms, large radiation reflectors, or aerodynamic flaps.[28–38]

The structural flexibility may interact with attitude control systems of spacecraft in a variety of ways. A NASA special report[39] focuses attention on the significance of this problem through a discussion of the anomalous behavior of several spacecraft. Besides citing useful references, it represents, with several others,[40,41] excellent review of the work in this area and puts current and anticipated problems in proper perspective. Furthermore, there are a few recommended procedures for analysis, simulation, and design. A paper by Noll et al.[42] effectively summarizes the NASA document. Experience to date suggests that in most cases problems arise not because of a lack of available analytical/numerical design procedures but because of failure on our part to recognize and appreciate the mechanism of the attitude control/structural flexibility interactions. The review papers just mentioned serve as an ideal introduction to the subject. They effectively summarize dynamical problems through illustration of several past and projected configurations where flexibility played or is likely to play an important role.

The problem of flexibility is neither of a recent origin nor is it restricted to spacecraft. Missile, aircraft, and surface vehicle designers have long been dealing with it. As pointed out by Ashley,[43] several of the techniques are directly applicable here. In many ways, however, spacecraft differ from other dynamical systems and require development of modified approaches specifically suited to the emerging field of astroelasticity.[44] However,

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* Reference 1 contains 132 entries related to librational dynamics of rigid satellites.

the fact remains that these problems are not new. They are old problems appearing in newer guise. They may give impressions of being foreign to the field in this form but essentially they represent synthesis of problems individually occurring in the field of structural dynamics, continuum mechanics, satellite mechanics, heat transfer, and control theory. Taken alone, in their degenerate form, the problems are usually solvable. However, in the synthesized form they attain complexity of a higher order but are, by no means, beyond our grasp. A vast body of literature exists, in the individual discipline, which can be drawn upon to tackle the problems. It is in this context that I consider flexibility problems tractable.

One often encounters repeated claims of having developed newer techniques to approach flexibility problems. Mostly they amount to application of well-known principles to problems that have acquired newer perspectives, or revised statements for well-established procedures. This in no way underestimates their contribution because, by and large, most engineering "research" falls in this category. Totally fresh innovations in techniques have been few—very few. On the other hand, it would be fair to say that available mathematical background coupled with digital and anolog simulation procedures are adequate to handle most of the existing problems of flexibility. They do not require "newer" approaches, as some investigators have asserted, but merely need application of established principles and techniques to a problem that is interdisciplinary in character.

A number of papers and reports dealing with the subject have poured out lately. The topics range from static studies of a simple component to dynamic simulations of very complex space stations. It is intended here to briefly review the relevant literature, familiarize with suggested procedures for analyzing flexible systems, summarize important conclusions, and comment on directions of future efforts that are likely to be fruitful. There is an implicit conviction here that "sound design analysis can increase the efficiency and lower the cost of the coming spacecraft, but the diverse specialists doing analysis must forge a common language to achieve these ends."[40] In all, nearly 200 papers are reviewed and loosely classified according to their scope of investigation which includes: 1) appendage analysis; 2) librational dynamics of gravity gradient systems with flexible appendages; 3) spinning systems with flexible appendages; and 4) librational control of flexible systems.

Pioneering contributions to the field are undoubtedly attributed to Likins,[2,40,45–56] Meirovitch[57–65] and Hughes[7,8,41,66–77] with indispensable mathematical support at a fundamental level from Pringle.[14,20,78–81] Likins' papers with their characteristic simplicity provide physical appreciation of the problem and have inspired extensive research activities of far-reaching importance. Precise modeling techniques, ingenious application of mathematics, and elegance of analysis are contributed by Meirovitch. Together with Likins, he has successfully laid a sound foundation for much of the work that has followed. Hughes' analyses cover a vast area. Besides presenting generalized formulation procedures, he has thrown considerable light on analysis and control of the system. A critical study of contributions by these researchers alone would provide a serious investigator sufficient background to the subject of flexibility effects in attitude dynamics of satellites, and open up a vast spectrum of challenging problems craving for solution.

A need for such an investigation was first felt when the anomalous behavior of Explorer-I,[82,83] Explorer-XX, and Alouette-I[67,69,84] was attributed to their flexible appendages. In general, the situation was aggravated by differential solar heating. Explorer-I was passively spin-stabilized about its principal axis of minimum moment of inertia—a configuration which later proved to be unstable. Thomson and Reiter,[83,85] followed by Meirovitch,[57] attributed this behavior to the flexible whip antennas. Alouette-I consisted of a compact central body with four long, flexible, extendable boom antennas. After successful launch and deployment, its spin gradually diminished from 1.5 rpm to near zero over the three-year span.[86] This unexpected rapid decay could not be explained by the mechanisms described earlier.[87] The analysis by Etkin and Hughes[67] revealed the cause to be an interaction of boom flexibility with solar radiation. A transverse temperature gradient resulted in bending of the antennas leading to an asymmetry in the spacecraft geometry, thus generating a despinning torque due to solar radiation pressure (for certain configurations the torque can be used to generate spin[88]). Hughes and Cherchas[68] extended the theory to include the effect of the Earth's shadow and removed the restriction on the spin vector to be normal to solar radiation. Vigneron[89] has considered these effects on the crossed-dipole system. The despin problem was resolved for the flight of Alouette II by mounting, at the ends of the modified booms, small metallic reflector plates which provided a compensating torque.

These analyses conclusively established that although gravitational, magnetic, and aerodynamic effects lead to errors in stationkeeping, an equally significant problem arises due to deformation of flexible appendages caused by solar radiation induced differential heating. The spacecraft motion as well as the deflections make the heat input a time-dependent function. Attempting to reach an equilibrium position, the booms oscillate. This self-excited motion driven by solar radiation may become appreciable due to a step input of heat while passing from the Earth's shadow into sunlight. The phenomenon was experienced by 1963-22A,[84] the first gravity stabilized spacecraft, which showed a resulting error of 10° from the local vertical. The use of highly reflective material for the booms on a subsequent flight (1963-49B[90]) limited the motion to less than 2.5°. The DODGE (Department Of Defence Gravity Experiment) checked the usefulness of gravity gradient at near synchronous altitudes.[91,92] The boom bending experiment[93] established the effectiveness of end-mass dampers in reducing the oscillations. The telemetry data from a number of other gravity-gradient satellites including the Naval Research Laboratory Satellites GGSE-III, IV, V, VI[94,95] and 0V1-10[96] strongly indicated the presence of "thermal flutter." As the booms were, in general, open section cylinders with an overlap (STEM-Self-storing Tubular Extensible Module[97]), numerous papers have appeared which investigate their thermoelastic and other relevant behavior.[98–128]

Of considerable interest is the analysis, by Augusti,[111] which studies the stability of elastic structures, modeled as one or two degree-of-freedom systems loaded by nonconservative forces, namely the radiant heat. It concludes that motion of the boom is stable if it is pointed towards the sun and unstable when oriented away from it. Jordon,[112] even with a wrong sign in the thermal curvature expression, came to the same conclusion. Subsequently, Yu[113] extended the analysis to the continuous beam case and inferred just the opposite. This was attributed to several unjustifiable simplifications.[114,115] Through a more rigorous analysis[116] Yu corrected his results and showed that when two cantilever-mode approximation is used, thermal instability can occur with the boom pointing towards or away from the sun.

The flight experience has led to many improvements in the boom design. The BI-STEM[125] (two STEMs placed one inside the other with their seams 180° apart), wire-screen booms,[126] and booms with "windows" in combination with selected thermal coating on both inside and outside surfaces[127] are among a few recent developments. Herzl[129] has presented an excellent review of the state-of-the art and anticipated future trends in their development.

These appendage analyses represent only a step in the study of satellite attitude dynamics and control. In its utmost generality, the problem indeed is quite challenging and not easily amenable to well-known procedures. Even exploration of a specific configuration would demand considerable ingenuity and enormous efforts. Concise formulations of the problem seem to represent an achievement of some measure.[19,21,25,43,45,49,50,72,130–137] It is therefore not surprising that, until recently, the literature has been devoid of comprehensive efforts at response and stability analysis of such systems.

In most cases, the spacecraft can be modeled in one of four ways:[40] 1) rigid body or assemblage of rigid bodies;

2) quasi-rigid bodies; 3) rigid body or assemblage of rigid bodies with flexible appendages; or 4) elastic bodies. The first category is, perhaps, the most comprehensive one as any complex structure can be discretized into a finite number of interconnected rigid bodies allowing a systematic development of equations. This discrete coordinate formulation[19,21,137] is limited only by the available computational capacity.

In the second case, a designer assumes short-term rigid body behavior and uses the resulting predictions of energy dissipation rate to obtain long-term secular estimates of vehicle motion. Though conceptually imprecise, the method has proved to be quite useful and is referred to by several names including the "energy-sink approach."[40–45,83]

The third group represents an idealization of spacecraft of the next generation. Using distributed or modal coordinates for the appendages, which are characterized as linearly elastic and subject to small deformations, and discrete coordinates describing the unrestrained motions of the rigid bodies, the procedure aims at evolving a "hybrid coordinate simulation."[50,51,131,132] Although this rather fancy terminology originated with Likins and his associates, the procedure itself was effectively employed, quite earlier, by Meirovitch and Nelson,[58] who also investigated the effect of series truncation.

Finally, the elastic systems can be analyzed through the normal-mode coordinates. The approach is quite well known and has been applied in the analysis of large spacecraft.[138,139] The use of "angular momentum principle"[130] and "adiabatic invariant strain tensor average" are among a number of other approaches being developed to study complex, fragile space structures.[140]

Effects of the thermoelastic behavior of booms on the pointing accuracy[141] and stabilization characteristics of the gravitationally oriented satellites can be quite significant. A digital-analog simulation[142] of the system has provided quantitative estimate of the resulting errors. The simulation included the effect of solar pressure—the main source of anomalous behavior. It was observed that the method of deployment of antennas has a significant effect on capture of a satellite by the gravity gradient and its subsequent dynamic behavior. Kanning's analysis[143] of a satellite with an arbitrary arrangement of booms suggests that changing geometry and mass distribution are significant particularly in the analysis of asymmetrical configurations. The analysis by Katucki and Moyer[144] supports the observation.

Paul[11,145] analyzed, through linearization, a simplified one-dimensional satellite in which the flexibility was intentionally included through a spring-mass-damper unit. This model, which approximated the first successful gravity-gradient satellite 1963-22A,[84] was extended by Buxton et al.[146] who used a longitudinal/torsional spring, and by Chobotov[147] accounting for nonlinearities in the planar equations of motion. Popov and Rutkovskii[148] derived the relations for the planar flexural vibrations of a gravitationally stabilized satellite system while the corresponding three-dimensional equations, during small deformation, were presented by Kharitonova.[149] Using the discrete coordinate formulation in conjunction with linearization, Reiter[150] showed that a significant range of inertia configurations, which is stable for a rigid body, is unstable when flexible, even with damping. Mitchell and Liu[151,152] noted a change in the librational frequency due to material elasticity and established the conditions for flexural resonances. Newton and Farrell[153] determined twelve pairs of complex conjugate eigenvalues and showed that a skewed damper, although capable of removing attitude librations, does not provide flexural stability.

Frueh and Miller[154,155] investigated the effect of elasticity on the performance of manned space stations. The work, however, did not contain any provision for gross rigid body motions. These were accounted for by Austin[156–158] who, in a more realistic approach, analyzed two axisymmetric rigid bodies connected in such a way as to permit relative motion about a common axis of symmetry. It was concluded that the effects of elasticity on gross rigid-body motion are of minor importance, at least for this model. Robe and Kane,[159] using rather simplified models of space stations, showed that certain vehicle configurations, which are predicted to be stable when analyzed as rigid, must be classed as unstable when flexibility is taken into account. With a proper choice of parameters the elastic attitude motion can be made to resemble that of the "associated rigid body." The foregoing, which forms only a beginning, suggests increasing interest in the field through recognition of the fact that high-frequency large-amplitude motions can occur for long, flexible satellites in near-circular orbits (RAE[160]) as well as compact satellites with flexible appendages in eccentric orbits (ATS-2[161]).

This brings us to an important question of stability of flexible configurations in the desired orientations. The structural flexibility may interact with spacecraft attitude control system necessitating a modified controller mechanism. Passive systems may be modeled as an integral part of the structure; however, active control systems require additional modeling to describe the dynamic character of the controller.[39] Several approaches to analyze and design controllers accounting for the flexibility have been proposed.[131,162–166] Digital simulations as well as analytical approaches[167–184] have been adopted to establish stability bounds. Unfortunately, complexity of the problem has limited most investigations to small amplitude motions.

Of fundamental interest is the stability of motion in the small, for a simplified spinning system, analyzed by Meirovitch and Nelson[58] where two flexible antennas were replaced by damped oscillators. This attempt at correlating system parameters with stability appears to be the first serious effort at exploring flexibility effects on satellite dynamics. Of equal importance is the study of stability in the large by Modi and Brereton.[185] Using the concept of invariant integral manifold, they obtained limiting conditions for planar librational stability of a slender, flexible satellite in an eccentric orbit and subjected to solar heating. The stability region was found to vary periodically with the solar aspect angle and, in general, was smaller in size compared to the corresponding rigid case.

Investigations in the field have attained an accelerated tempo in recent times with a large number of important papers. Meirovitch[60] extended application of the Liapunov's direct method to stability analysis of a system of hybrid differential equations associated with rigid spinning body containing elastic parts. Use of the variational principle in formulation of the problem and the Hamiltonian as a Liapunov function leads to a rather attractive approach producing "sharper stability criteria." Gale and Likins[51] examined the influence of flexible appendages on dual-spin spacecraft dynamics and control. Linearized equations provide the basis for digital simulation and for stability as long as the appendages have at least a minimal amount of structural damping. Using a hybrid coordinate formulation, Likins and Fleischer[52] have proposed a three-stage process for a flexible vehicle's attitude control design: 1) single axis response of a linearized system; 2) eigenvalue analysis of the coupled linear system; 3) nonlinear analysis through numerical integration. Budynas and Poli[186] analyzed the planar stability of a rigid body containing two flexible antennas located at 180° from each other and in the orbital plane. The conditions for stability in the small are found to include the well-known rigid body criterion and, in addition, requirements on the coupled rigid-elastic motion.

The application of Liapunov's Direct Method was further extended by Meirovitch and Calico[63] to the hybrid system of equations in which test density functions are not readily defined. The authors illustrate the application of the method through a system comprising a torque-free spinning rigid body with three pairs of rigidly attached flexible rods. Kulla[187] studied the linearized system of equations describing a spinning body with elastic rods along and perpendicular to the spin axis. The impedance of the rods was used to evaluate the dynamics of the system in terms of six degrees-of-freedom of the central body. The analysis suggests that arresting the translational motion of the center of mass leads to a reduction in stability. Of considerable significance is the concept of multiple control as proposed by Porcelli[188] where a primary loop controls the main portion of the structure while auxiliary loops actively damp the flexible appendages. Tsuchiya and Saito[189] have examined the dynamics of spin-stabilized satellites with flexible appendages nominally in the spin plane through a modification of the energy sink method by an averaging approach. A closed-form solution is obtained for the large damping of the nutational motion during the nonlinear resonance, the phenomenon first studied, numerically, by Pringle.[14] Constrained and unconstrained modes of vibration and their application in the dynamical investigation of flexible satellites are explored by Hughes et al.[8,72,73] It is observed that fewer unconstrained modes are required to adequately represent the flexibility of the spacecraft. The authors conclude that for most cases, the techniques of continuum mechanics can be employed to deal with small deflections and in the design of a reliable attitude control system. A paper by Hughes[7] studies characteristic modes of twist/pitch oscillations associated with a large, flexible solar panel. The author has also indicated the procedure for including the flexibility effects through a modified control system simulation. Following a similar approach, Cherchas[190] has obtained unconstrained modes of vibration involving body pitch and array twist. Importance of an offset between the array center line and the spacecraft center of mass leading to array bending modes is pointed out.

Meirovitch and Calico[64] have presented a comparative study of three different approaches to Liapunov stability analysis of hybrid dynamical systems associated with flexible satellites: the method based on testing density functions, the method of integral coordinates, and the modal analysis. It was observed that the first two methods lead to relatively simple closed-form stability conditions in terms of system parameters while the third method of modal analysis, in general, yields more involved criteria depending on the number of modes used to represent the elastic displacements. Attitude stability and nutation decay times for a dual-spin satellite with a large flexible solar array were investigated by Cherchas and Hughes.[74] It was found that when the solar array is considered rigid, the ratio of rotor inertia to the geometric mean of the vehicle transverse inertias must be greater than unity for stability unless there is energy dissipation in the despun section. A recent study by Keat[133] on attitude stabilization by reaction jets, of a satellite with gravity gradient booms and passive damper, takes into account the nonrigidity of the system. Essential contribution of the paper lies in the use of Hamiltonian to develop the control laws. Teixeira-Filho and Kane[191] developed a method for constructing stability criteria applicable to spinning motion of torque-free, elastic, dissipative systems possessing a finite number of degrees of freedom. An energy function modified to take angular momentum integrals into account was employed to construct a Liapunov function. Application of the method was illustrated through an example of a spinning, rigid satellite with four elastically mounted antennas. A linearized analysis of spin stabilization using four radial wire antennas with tip masses, the configuration corresponding to IMP-J and IME spacecraft, is presented by Longman and Fedor.[192] The differential equations for the response of each of the fourteen modes of vibration are solved analytically in terms of Bessel functions, a Struve function, and standard trigonometric functions. Approximate solutions are also obtained using the WKB method in conjunction with an asymptotic expansion.

It would be appropriate to briefly comment on the papers presented at the recent AAS/AIAA Astrodynamics Conference[55,65,193–195] and the XXIVth Congress of the International Astronautical Federation.[196–200] Modi and Kumar[193] examined librational dynamics of gravity-oriented satellites with an arbitrary number of large flexible appendages deforming under differential solar heating. Both the system response and stability in the large were studied using a quasi-steady representation of the elastic mode. An important aspect of the analysis is the fact that the results can be presented in terms of nondimensional flexibility parameters which do not involve explicit specification of the number or the physical character of the flexible members present. Meirovitch[65] studied the Liapunov stability of a hybrid dynamical system in the neighborhood of a nontrivial equilibrium. The theory was applied to a gravity-gradient stabilized satellite with flexible appendages. Philosophical discourse on the modal and finite element approaches in analyzing librational dynamics of a spinning system with flexible radial and axial members formed the subject of a paper by Likins et al.[55] It was shown that realistic assumptions concerning elasticity can significantly reduce computational efforts without substantially affecting the accuracy of end results. Fleischer[194] has developed a computer subroutine which solves the attitude equations of motion for any vehicle idealized as a topological tree of hinge-connected rigid bodies. It was used to simulate and analyze interaction between scientific instrumentation's control system and the flexible Mariner Venus/Mercury spacecraft. Rather interesting and of considerable significance is a paper by Larson[195] which addresses to the problem of stochastic optimal control of an n-body spacecraft. The controller uses angular measurements associated with the base body to obtain smoothed estimates of the entire state vector. It can be easily implemented and promises a significant improvement in the system performance.

Popp and Schiehlen[196] have discussed the degree of performance optimality as referred to wobble damping systems, and its application to a spinning flexible spacecraft modeled as a central rigid body with four flexible booms. The authors come to the well-known conclusion that an active system which uses additional information of the state of the spacecraft leads to an improvement in the degree of optimality. General equations of motion for a flexible satellite are presented by Bianco et al.[197] who also comment on their damped oscillations around the equilibrium configuration and application of Liapunov's first method in the stability study. Of considerable interest are the studies by Huang and Das[198,199] concerning singular perturbation analysis of flexible satellites and their thermoelastic flutter models. The investigations form a part of a comprehensive and well-organized plan of study involving pointing error analysis of geostationary satellites. Pfeiffer and Pohl[200] have presented a linearized analysis of a rather interesting configuration-geostationary satellite with a rigidly mounted flywheel, a damper and a pair of flexible solar arrays. Vibration of continuous appendages is represented by assumed modes. The analysis suggests that roll-yaw and pitch motions are decoupled. Moreover, roll-yaw motion is coupled only with the asymmetric bending vibrations of the panel and pitch motion has coupling only with the symmetric bending and torsion. The enlargement of the limit-cycles in the flexible case clearly suggests a deterioration in stability, primarily affected by the angular momentum of the flywheel, moments of inertia of the satellite and panel stiffness.

A word of caution concerning the validity of planar analyses as presented by several authors would be appropriate. For rigid axisymmetric satellites in the gravity gradient field, inplane disturbances do not excite out of plane motion (the converse is not true)[30,201] and, hence, pure planar librations are indeed possible in highly specialized situations. Furthermore, as observed by Modi and Brereton,[202] the planar analysis occasionally leads to conservative estimates of stability in the large. However, for flexible systems even such preliminary suggestions may turn out to be misleading. Although Modi and Kumar's[193] comparison of planar and three-dimensional data showed occasional similarity, any such appearance of agreement should not be considered as suggesting trend. One is thus faced with a perennial question of what is desirable—approximate analysis of the general motion or a relatively precise analysis of a constrained system.

Let me close with a few general remarks concerning desirable trends for future efforts.

1) More attention should be directed towards evolution of general analyses rather than exploration of specific configurations.

2) Flexibility affects dissipation as well as the instantaneous inertia character of a system. Several authors have suggested

that these changes can be accommodated by introducing "effective moments of inertia." The concept, though attractive from design office consideration, is misleading and in general will lead to erroneous conclusions. After all the problem is more involved and any impulse to provide over simplified solutions, however attractive they may appear, should be resisted until more precise analyses are available. It is only with better understanding of the system that one is likely to be successful in evolving simpler, workable models.

3) Flexibility interacts with control system while control forces affect flexible configurations. Thus, the problem is inherently conjugate in character and should be treated as such.

4) As general approach to the problem gets well-established, details will start to receive more attention, e.g., quasi-steady, discrete or continuous representation of elastic appendages, modes to be used from accuracy and computational considerations, etc. In this context, consideration should also be given to material, structural, configurational, and support studies to minimize transmission of flexibility effects to a stabilized platform.

5) Complex character of the problem has limited most analyses to small deformations and librations. Dynamic response and stability analysis in the large is, of course, formidable but promises to be equally exciting and rewarding.

6) There is a considerable scope for contribution to the problem of forced flexible systems.

7) Yet another problem, which is being discussed with a degree of urgency and is definitely going to attract more attention, is that of space power generation using solar energy. Several different concepts and configurations have been proposed[203,204] but the one that has received considerable attention is that due to Glaser.[205] The multibody configuration, comprising of a gigantic solar collector connected to a microwave transmitter by a few miles long tether, presents a host of problems of flexibility interaction with attitude dynamics and control.

The problem of flexible body dynamics is here with us to stay. This brief review of the literature is by no means complete; however, it is sufficiently comprehensive to familiarize a serious investigator with the challenging field.

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